Folding

$$
\begin{array}{ccc}
: & & f \\
/\ \backslash & & /\ \backslash \\
a_0 \quad : & & a_0 \quad f \\
\quad /\ \backslash & & \quad /\ \backslash \\
\quad a_1 \quad : & & \quad a_1 \quad f \\
\qquad /\ \backslash & & \qquad /\ \backslash \\
\qquad a_2 \quad : & & \qquad a_2 \quad f \\
\qquad\quad /\ \backslash & & \qquad\quad /\ \backslash \\
\qquad\quad a_3 \ [\ ] & & \qquad\quad a_3 \quad e
\end{array}
$$
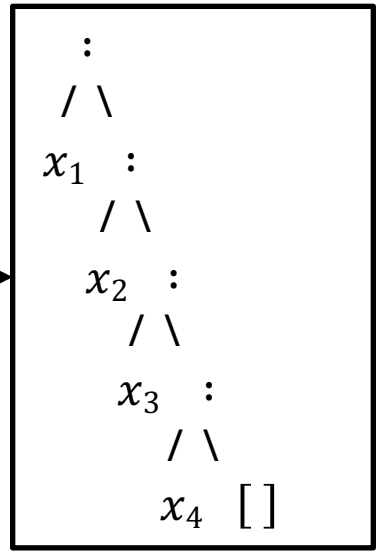
CHEAT-SHEET

#5

folding a list **right** and **left** using **Cons** and **Nil** results in the **identity** and **reverse** functions

$(:) = \lambda x.\lambda y. x : y$
$(\cdot\cdot) = \lambda x.\lambda y. y : x$
$(\cdot\cdot) = flip \ (:)$
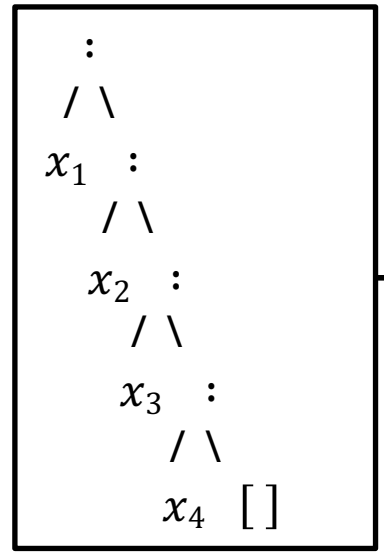$\qquad where \ flip \ f \ x \ y = f \ y \ x$

$foldr \qquad\qquad :: (\alpha \to \beta \to \beta) \to \beta \to [\alpha] \to \beta$
$foldr \ f \ e \ [\,] \qquad = e$
$foldr \ f \ e \ (x{:}xs) \quad = f \ x \ (foldr \ f \ e \ xs)$

replace:
$(:)$ with f
$[\,]$ with e

$xs = [x_1, x_2, x_3, x_4]$

replace:
$(:)$ with $(:)$
$[\,]$ with $[\,]$

$foldr \ (:) \ [\,] \ xs$

$\underbrace{\qquad\qquad}_{identity}$

$reverse$

$foldl \ (\cdot\cdot) \ [\,] \ xs$

var $acc = [\,]$
foreach$(x \ in \ xs)$
$\quad acc = acc \cdot\cdot x)$
return acc

$foldr \ (:) \ [\,] = identity = \ foldr \ (\lambda x.\lambda y. x : y) \ [\,]$
$foldl \ (\cdot\cdot) \ [\,] = reverse = \ foldl \ (\lambda x.\lambda y. y : x) \ [\,]$

```
> id  = foldr (:) []
> rev = foldl (flip (:)) []
> id [1,2,3,4]
[1,2,3,4]
> rev [1,2,3,4]
[4,3,2,1]
```
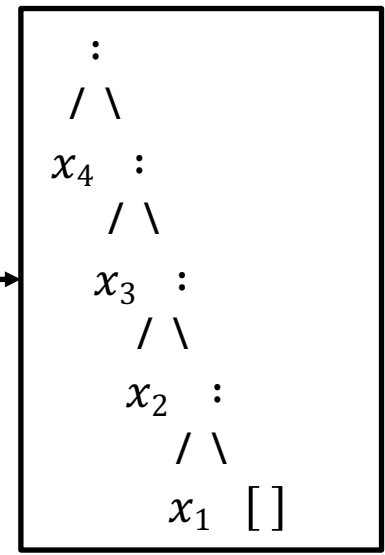
```
> id  = foldr (\x y -> x:y) []
> rev = foldl (\x y -> y:x) []
> id [1,2,3,4]
[1,2,3,4]
> rev [1,2,3,4]
[4,3,2,1]
```

$[x_1, x_2, x_3, x_4]$
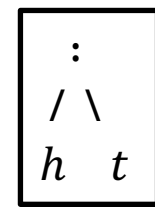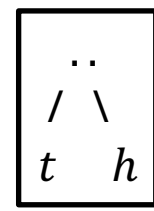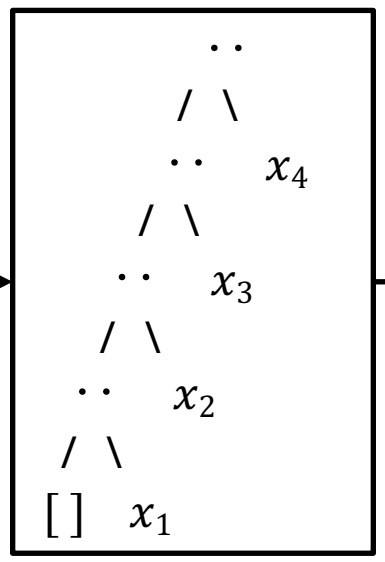
$[x_4, \ x_3, \ x_2, \ x_1]$

$x_1{:}(x_2{:}(x_3{:}(x_4{:}[\,])))$

$foldl \qquad\qquad :: (\beta \to \alpha \to \beta) \to \beta \to [\alpha] \to \beta$
$foldl \ f \ e \ [\,] \qquad = e$
$foldl \ f \ e \ (x{:}xs) = foldl \ f \ (f \ e \ x) \ xs$

var $acc = e$
foreach$(x \ in \ xs)$
$\quad acc = f(acc, x)$
return acc

$((([\,] \cdot\cdot x_1) \cdot\cdot x_2) \cdot\cdot x_3) \cdot\cdot x_4$

equivalent to

$x_4 : (x_3 : (x_2 : (x_1 : [\,])))$